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GOPAL GOVIND POY RAITURCAR COLLEGE OF COMMERCE AND ECONOMICS

FARMAGUDI, PONDA - GOA

B.C.A. UGC-CCFUP (SEMESTER-I) REGULAR EXAMINATION, October 2024

MINOR-1 MAT-111 ELEMENTARY MATHEMATICS

Duration: 2 Hrs **Marks:80**

Instructions:

1. All questions are compulsory.
2. Figures to the right indicate full marks.
3. Use of non-scientific calculators is allowed.

Q1) Answer **each** of the following**:** **(8×2=16)**

i. Define ordered pair and cartesian product. **(BL1,CO2)**

ii. Given $z_1 = 5 + 9i$ and $z_2 = 8 - 4i$, find Z_1Z_2. **(BL1,CO4)**

iii. Given $f(x) = 2x + 1$ and $g(x) = 10x + 2$ find $(g \circ f)$ **(BL1,CO2)**

iv. Write the truth table for 'IF THEN'. **(BL1,CO1)**

v. If $U = \{x \ \epsilon \ \mathbb{N} | x \leq 16\}$, $A = \{2,3,5,9,10,12,15\}$and $B = \{3,5,6,10,12,15,16\}$ then find $A \cup B$ and $A \cap B$. **(BL1,CO2)**

vi. Define the order and degree of differential equation. **(BL1,CO6)**

vii. Write an example of a one-one relation and a many one relation. **(BL1,CO2)**

viii. Write the conjugate of complex number $z = x + yi$, and hence write the conjugate of $z_1 = 12 - 9i$. **(BL1,CO4)**

Q2) A) i) Prove **both** the De' Morgans Laws using $U = \{x \ \epsilon \ \mathbb{N} \mid x \leq 12\}$as the universal set, $A = \{1,3,5,9,10\}$ and $B = \{2,4,5,9,10,12\}$. **(BL2,CO2) (04)**

ii) Simplify $\dfrac{6+2i}{8-9i}$. **(BL2,CO4) (02)**

OR

Q2) A) iii) If $f(x) = ax^2 + b,\ f(1) = 6$ and $f(2) = 12$. Find a and b . **(BL2,CO2) (04)**

iv) Find the dot product $A \cdot B$, if $A = 12\hat{i} + 5\hat{j} - 6\hat{k}$ and $B = -3\hat{i} + 9\hat{j} - 10\hat{k}$ **(BL2,CO3)(02)**

Q2) B) i) Discuss the continuity of the function $f(x) = \begin{cases} \frac{x^2+2x-24}{x-4} \ ; \ x \neq 4 \\ 12 \ ; \ x = 4 \end{cases}$. If the function is discontinuous, state the type of discontinuity and make the function continuous. **(BL2,CO3) (06)**

ii) Verify that $y = x^3 + x^2 + 6$ is a solution of the differential equation
$\frac{1}{(3x^2 + 2x)} \frac{dy}{dx} - 1 = 0.$ **(BL2,CO6) (04)**

Q3) A) i) Let $f : \mathbb{R} \to \mathbb{R}$ be defined by $f(x) = 5x - 9$. Show that f is one-one and onto and find its inverse. **(BL3,CO2) (04)**

ii) Find $(f + g)(x)$ if $f(x) = x^2 - 4x + 9$ and $g(x) = 6x^3 - 9x^2 + 5x + 10$. **(BL3,CO3) (02)**

OR

Q3) A) iii) Let $f : \mathbb{R} \to \mathbb{R}$ be defined by $f(x) = \frac{4}{9}x + 6$. Show that f is one-one and onto and find its inverse. **(BL3,CO2) (04)**

iv) Find $f(x) \cdot g(x)$ if $f(x) = \frac{12}{5}x + 9$ and $g(x) = 5x - 15$. **(BL3,CO3) (02)**

Q3) B) i) Find the Curl of the vector $\vec{f} = (x^2yz, x + 12y^2 - Z, xyz^3)$. **(BL3,CO5) (06)**

ii) Find the divergence of the vector $\vec{g} = (x^2 - 15x + 4{,}5y^3 + 8x - 10{,}z^4 - 9z^2$. **(BL3,CO5) (04)**

Q4) A) i) a) If $A = (2,4,6)$, $B = (3, -9,2)$ and $C = (-4,10,12)$, find $3A - 2B + C$. **(BL4,CO5) (02)**

b) Find the dot product $A \cdot B$ if $A = (6,9,1)$ and $B = (-5,7,2)$. **(BL4,CO5) (02)**

ii) Differentiate $f(x) = x^2 logx$ **(BL4,CO3) (02)**

OR

Q4) A) iii) Find the cross product of the vectors $\overrightarrow{A} = 2\hat{i} - 6\hat{j} + 5\hat{k}$ and $\overrightarrow{B} = -\hat{i} + 2\hat{j} + 8\hat{k}$. **(BL4,CO5) (04)**

iv) Compute the values of $g(x)$ at $x = 9{,}6$ if $g(x) = \frac{4}{5}x^2 - 10x + 19$. **(BL4,CO2) (02)**

Q4) B) i) a) Compute $A' \cup (B \cap C)'$ if $A = \{2,3,5,6,8\}$, $B = \{1,2,6,9,10\}$ and $C = \{3,4,6,9,10\}$, and the universal set is $U = \{x \in \mathbb{N} \mid x \leq 10\}$ **(BL4,CO2) (03)**

b) Transform the following statement into symbolic form and construct it's truth table.
"I will learn guitar if and only if I have the passion for music". **(BL4,CO2) (03)**

ii) Prove that the following statements are logically equivalent.
$p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$ **(BL4,CO1) (06)**

Q5) A) i) Solve the following differential equation:
$(2y^3 - 6y + 8)\frac{dy}{dx} - 9x^2 - 7x = 0$ **(BL4,CO6) (03)**

ii) Evaluate $\lim_{x \to -9} \frac{x^2 + 4x - 45}{x + 9}$ **(BL4,CO3) (03)**

OR

Q5) A) iii) Form the differential equation representing the family of curves $y = ax^2 + bx$, where a and b are constants. **(BL4,CO6) (03)**

iv) Find the particular solution of the differential equation $\frac{dy}{dx} - (2x^2 + 5x) = 0$,

Satisfying the initial condition $y(1) = 7$ if the general solution is $y(x) = \frac{2}{3}x^3 + \frac{5}{2}x^2 + c$

(BL4,CO3) (03)

Q5) B) i) Represent the complex number $z = 9 + 6i$ in the polar form. $[tan^{-1}(\frac{6}{9}) = 0.59]$. **(BL3,CO4) (04)**

ii) Construct truth table for the compound statement $\sim (p \wedge p) \rightarrow (\sim q \vee p)$. **(BL3,CO1) (04)**

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