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GOPAL GOVIND POY RAITURCAR COLLEGE OF COMMERCE AND ECONOMICS

FARMAGUDI, PONDA - GOA

B.C.A. UGC-CCFUP (SEMESTER-I) REGULAR EXAMINATION, NOVEMBER 2023

MINOR-1 MAT-111 ELEMENTARY MATHEMATICS

Duration: 2 Hrs **Marks:80**

Instructions:

1. All questions are compulsory.
2. Figures to the right indicate full marks.

Q1) Answer **each** of the following**:** **(8×2=16)**

i. Define Equivalence relation. **(BL1,CO2)**

ii. Given $Z_1 = 6 - 5i$ and $Z_2 = 4i - 3$, find $Z_1 Z_2$. **(BL1,CO4)**

iii. Given $f(x) = \frac{x+2}{x-1}$, show that $f\big(f(x)\big) = x$. **(BL1,CO2)**

iv. Define Negation and construct truth table for Negation. **(BL1,CO1)**

v. If $U = \{x: x \in \mathbb{N}; x \leq 13\}$, $A = \{1,3,5,7,9,11,13\}$ and $B = \{3,4,5,6,7\}$ then find $A \cap B$ and $A' \cup B$. **(BL1,CO2)**

vi. Define order and degree of differential equation. **(BL1,CO6)**

vii. Give an example of a relation that is reflexive but neither symmetric nor transitive. **(BL1,CO2)**

viii. Define conjugate of complex number and evaluate $6 - 3i + \overline{6 - 3i}$. **(BL1,CO4)**

Q2) A) i) Prove **both** the De' Morgans Laws using $U = \{x: x \in \mathbb{N}, 1 \leq x \leq 13\}$ as the universal set, $A = \{1,2,3,4,5,6\}$ and $B = \{1,3,5,7,9,11,13\}$. **(BL2,CO2) (04)**

ii) Simplify $\frac{4+3i}{3-4i} + \frac{3-4i}{4+3i}$. **(BL2,CO4) (02)**

OR

Q2) A) iii) If $f(x) = ax^2 + bx + 2$ and $f(1) = 3$ and $f(4) = 42$. Find a and b. **(BL2,CO2) (04)**

iv) If the function $f(x) = \begin{cases} \frac{x^2-25}{x-5}, if\ x \neq 5 \\ C, \quad if\ x = 5 \end{cases}$ is continuous what is the value of C. **(BL2,CO3)(02)**

Q2) B) i) Discuss the continuity of the function $f(x) = \begin{cases} \frac{x^2-9x+14}{x^2+x-6} & ; \quad x \neq 2 \\ 5 & ; \quad x = 2 \end{cases}$. If the function is discontinuous, state the type of discontinuity and redefine the function. **(BL2,CO3) (06)**

P.T.O.

ii) Verify that $y = ae^{2x} + bxe^{2x}$ is a solution of the differential equation $\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 4y = 0.$ **(BL2,CO6) (04)**

Q3) A) i) Let $f: \mathbb{R} \to \mathbb{R}$ be defined by $f(x) = 3x - 7$. Show that f is bijective and find its inverse. **(BL3,CO2) (04)**

ii) Find values of x for which function $f(x) = x^3 - 12x + 9$ is decreasing. **(BL3,CO3) (02)**

OR

Q3) A) iii) Prove that the relation R on the set of integers $\mathbb{Z}$ defined as $R = \{(x, y): x - y \; is \; divisible \; by \; 3; x \in \mathbb{Z}, y \in \mathbb{Z}\}$ is an equivalence relation. **(BL3,CO2) (04)**

iv) Find the area of the parallelogram whose adjacent sides are given by vectors $\vec{a} = \hat{\imath} - 2\hat{\jmath} + 4\hat{k}$ and $\vec{b} = 3\hat{\imath} - \hat{\jmath} + 2\hat{k}$. **(BL3,CO5) (02)**

Q3) B) i) For the given vector field $\vec{V} = xy^2\hat{\imath} + x^2yz\hat{\jmath} - 3y^2z\hat{k}$, find

a) Divergence of $\vec{V}$.

b) Curl of $\vec{V}$. **(BL3,CO5) (06)**

ii) Find maximum and minimum values of the function $f(x) = x^3 - 6x^2 + 9x - 7$. **(BL3,CO3) (04)**

Q4) A) i) Show that the three points whose position vectors are $A = (-2,3,5), B = (1,2,3),$ $C = (7,0,-1)$ are collinear. **(BL4,CO5) (04)**

ii) Differentiate $y = 4x^2 log x$ with respect to x. **(BL4,CO3) (02)**

OR

Q4) A) iii) Find a unit vector perpendicular to both the vectors $\vec{a} = 3\hat{\imath} + 2\hat{\jmath} - \hat{k}$ and $\vec{b} = 3\hat{\imath} + 4\hat{\jmath} - 3\hat{k}$. **(BL4,CO5) (04)**

iv) Let $A = \{1,2,3,4\}$ and a relation on A be $R = \{(1,1), (1,2), (2,1), (2,2), (2,3), (3,2), (3,3)\}$. Prove that the relation R is symmetric but not reflexive. **(BL4,CO2) (02)**

Q4) B) i) In a group of 150 people of certain locality, the number of people reading newspapers 'The Navhind Times', 'Herald' and 'Gomantak Times' are 50, 40, 47 respectively. 15 read both Navhind Times and Herald, 14 read Herald and Gomantak Times, 13 read Gomantak Times and Navhind Times, 5 read all the newspapers. Find

a) Number of people who read neither of the newspapers.

b) Number of people who read Navhind Times or Herald or both.

c) Number of people who read Herald or Gomantak times or both. **(BL4,CO2) (06)**

P.T.O.

ii) Prove that the following statements are logically equivalent.

$p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$ **(BL4,CO1) (06)**

Q5) A) i) Solve the following differential equations:

$e^x(3x^2 + 1)dx + e^x(2 - y)dy = 0$ **(BL4,CO6) (03)**

ii) Evaluate $\lim_{x \to 2} \left[\frac{x^2-3x+2}{(x-2)}\right]$ **(BL4,CO3) (03)**

OR

Q5) A) iii) Form the differential equation representing the family of curves $y = \text{a} \sin (x + b)$, where a, b are constants. **(BL4,CO6) (03)**

iv) Divide 10 into two parts such that the sum of twice of one part and square of the other is a minimum. Find the two parts. **(BL4,CO3) (03)**

Q5) B) i) Express the complex number $(\sqrt{3} + i)$ in polar form. **(BL3,CO4) (04)**

ii) Construct truth table for the compound statement $\sim(p \wedge q) \longrightarrow \sim p \vee \sim q$. **(BL3,CO1) (04)**

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