# Goa Vidyaprasarak Mandal's <br> Gopal Govind Poy Raiturcar College of Commerce and Economics Ponda- Goa <br> B.C.A. (Semester - II) Supplementary Examination, May/June 2017 204 DISCRETE MATHEMATICS 

Duration : 2 Hrs
Marks : 50

## Instructions:

I. All the questions are compulsory however internal choices are given.
II. Use of calculators is not allowed.
III. Marks to the right indicate full marks.
Q.1. Fill in the following blanks:
i. A statement for which all the truth values are true is called a $\qquad$
ii. $(72)_{10}$ is equivalent to $\qquad$ in binary form.
iii. Involution law on Boolean algebra is $\qquad$
iv. Let $X=\{n \mid n$ is a natural number $\}, A=\{2 n \mid n$ is a natural number $\}$.

Then complement of $\mathrm{A}\left(\mathrm{A}^{\mathrm{c}}\right)=$ $\qquad$
v. Base for the hexadecimal system is
vi. Let $f$ be a bijection. Then $f^{l}(f(x))=$
vii. The number of possible arrangements of the letters of the word 'SCIENCE' is $\qquad$
viii. If $f(x)=x+1$ and $g(x)=\mathrm{e}^{\mathrm{x}}$ then $f \cdot g(x)=$ $\qquad$
ix. Length of the string 'aaaabbcbbddd' is $\qquad$
x. ${ }^{5} \mathrm{C}_{3}=$ $\qquad$

## Q.2.

i. Check whether $(\mathrm{p} \wedge \wedge q) \Lambda \rightarrow \sim p$ is a contradiction.
(3)
ii. Write $(\mathrm{p} \rightarrow r) \vee \mathrm{q}$ for the following statements
$\mathrm{p}: \mathrm{n}$ is a an odd number $\mathrm{q}: \mathrm{n}$ is a composite number, r : n is not divisible by 5 .
iii. What are the symbols for OR and AND gates?

Draw truth tables for OR and AND gates to show the outputs for the possible inputs.

## OR

## Q.II

i. If x is even then 2 divides x .

Write inverse and contrapositive of the above implication.
ii. Show that $(p \uparrow q) \oplus(p \uparrow q)$ is a contradiction.
iii. Show that $(a+b) \cdot(\bar{b}+c)+b \cdot(\bar{a}+\bar{c})=a \cdot \bar{b}+a \cdot c+b$.

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## Q.3.

i. Convert $(101101)_{2}$ to hexadecimal number system.
ii. Find the coefficient of $x^{2} y^{3}$ in the expansion of $(2 x+y)^{5}$.
iii. Let $\mathrm{A}=\{1,2,4,7,8\}$ and $\mathrm{R}=\{(\mathrm{a}, \mathrm{b}) \mid \mathrm{a} \geq \mathrm{b}\}$.

What are the elements in R ? Is R a partially ordered set? Justify

## OR

## Q.III.

i. Convert $(0.625)_{10}$ to binary form.
ii. Using binomial theorem show that $\sum_{k=0}^{n}\binom{n}{k}(-1)^{\mathrm{k}}=0 \cdot\left(\binom{n}{k}={ }^{\mathrm{n}} \mathrm{C}_{\mathrm{k}}\right)$
iii. Let $\mathrm{A}=\{3,6,9,12,15,18\}$ and relation R on A be given by
$\mathrm{R}=\{(\mathrm{a}, \mathrm{b}) \mid$ a divides b$\}$. What are the elements of R ? Is R an equivalence
Relation? Justify.

## Q.4.

i. Let $X=\{1,2,3,4,5,6,7,8,9,10\}, A=\{1,2,3,4,7,9\}$ and $B=\{1,2,6,7,8\}$,

Verify $(A \cap B)^{c}=A^{c} \cup B^{c}$.
ii. In how many ways 4 ladies and 6 gentlemen can be arranged in a row such that no two ladies are together?
iii. Find output for the following

iv. Let $G=\{N, V, \boldsymbol{\sigma}, P\}$, where $N=\{S, Q, R\}$ (with $S$ as the starting point $\}$, $\mathrm{V}=\boldsymbol{\sigma}=\{\mathrm{a}, \mathrm{b}, \mathrm{c}\}$ and $\mathrm{P}=\{\mathrm{S} \rightarrow \mathrm{a}, \mathrm{Q} \rightarrow \mathrm{aQ}, \mathrm{Q} \rightarrow \mathrm{cR}, \mathrm{R} \rightarrow \mathrm{b}\}$.
What is the language generated by G ?

## OR

## Q.IV.

i. Let $A=\{3,4,7,9,10\}, B=\{1,2,4,5,6,7\}$ and $C=\{1,5,6,7,9,10\}$

Find $(A \cap B) \cup C$.
ii. How many arrangements of the letters of the word 'MATHEMATICS' are possible?
iii. Simplify the Boolean expression $(x+y) \cdot \overline{(x+y})+(x+(\bar{x} \cdot y))$.
iv. Consider the regular expression $\mathrm{R}=a b^{*}(c \mid d)$.

Give any five distinct strings belonging to the language generated by the above language.

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## Q.5.

i. Using principle of mathematical induction show that the sum of squares of first $n$ natural numbers is $\frac{n(n+1)(2 n+1)}{6}$.
ii. Let $f: \mathbf{R} \rightarrow \mathbf{R}, f(\mathrm{x})=9 \mathrm{x}+4$. Is $f$ injective? Justify.
iii. A statistician conducted a survey of 500 people and found that 300 liked brand A and 270 liked brand B. He concluded that at least 70 people liked both brands A and B. Do you agree with him? Justify the answer.

## OR

Q.V.
i. Show that $1^{3}+2^{3}+\ldots \ldots . .+n^{3}=\left(\frac{n(n+1)}{2}\right)^{2}$.
ii. Let $f: \mathbf{R} \rightarrow \mathbf{R}, f(\mathrm{x})=4 \mathrm{x}-5$. Is $f$ surjective? Justify.
iii. A town has a total population of 4000 out of which 400 people own cars, 1000 people own bicycles and 300 own both cars and bicycles. How many in the town do not own either?

