

Goa Vidyaprasarak Mandal's
GOPAL GOVIND POY RAITURCAN COLLEGE OF COMMERCE AND ECONOMICS,
PONDA-GOA

B.C.A (SEMESTER-II) EXAMINATION, APRIL 2016

DISCRETE MATHEMATICS- BCA 204

Duration: 2 hours

Marks: 50

Instructions

- i. All the questions are compulsory however internal choices are given
- ii. Marks to the right indicate full marks

Q.1 Fill in the blanks

(1 × 10 = 10)

- a. is the logic symbol for NEGATION
- b. Define string over an alphabet σ
- c. Define the term Regular Expression
- d. Let A be a Boolean algebra, then $x \cdot 1 = \dots$ for all $x \in A$
- e. Let U be a universal set and let B be any subset of U then $B \cap U = \dots$
- f. The middle term of $(2x + 3)^4$ is
- g. ${}^n C_0 + {}^n C_1 + {}^n C_2 + \dots + {}^n C_{n-1} + {}^n C_n = \dots$
- h. Let $A = \{1, 2, 3, 4, 5, 6\}$ and $B = \{2, 4, 6\}$ then $A - B = \dots$
- i. Let $C = \{a, b, c\}$ and $D = \{2, 4, 6\}$ then $C \times D = \dots$
- j. Let $f(x) = x^2$ and $g(x) = \frac{1}{x}$ then $(f \circ g)(x) = \dots$

Q.2

- a. Convert the number $(11001110)_2$ from binary to decimal form (2)
- b. Use principle of mathematical induction to prove that
$$1^2 + 2^2 + 3^2 + \dots + n^2 = \frac{n(n+1)(2n+1)}{6}$$
 (3)
- c. Prove that the statement $(p \vee q) \wedge (p \downarrow q)$ is a contradiction (5)

OR

Q.II

- A. Convert the number $(3FED)_{16}$ from hexadecimal to decimal form (2)

B. Use principle of mathematical induction to prove that (3)

$$1 \cdot 3 + 2 \cdot 4 + 3 \cdot 5 + \dots + n(n+2) = \frac{n(n+1)(2n+7)}{6}$$

C. Prove that the statement $(p \wedge q) \rightarrow (p \vee q)$ is a tautology (5)

Q.3

a. Convert $(728)_{10}$ from decimal to octal from. (2)

b. Prove that the relation R on the set of integers Z defined as

$$R = \{(x, y) : x - y \text{ is divisible by } 5, x \in \mathbb{Z}, y \in \mathbb{Z}\} \text{ is an equivalence relation. (4)}$$

c. In a group of 150 people of a certain locality, the number of people reading newspapers 'The Navhind Times', 'Herald', and 'Gomantak Times' are 50, 40, 47 respectively. 15 read both 'The Navhind Times' and 'Herald', 14 read 'Herald' and 'Gomantak Times', 13 read 'The Navhind Times' and 'Gomantak Times', 5 read all the three newspapers. Find the number of people who read neither of the newspapers. (4)

OR

Q. III

A. Convert $(345)_8$ from octal to decimal from (2)

B. What is a partial order relation? Determine whether the relation R is a partial order relation on the set of positive integers

$$xRy \quad \text{iff} \quad x \geq y \quad (4)$$

C. 'Set of all natural numbers less than 20 that are divisible by 3'

Write the above set in both roster and set builder form. (4)

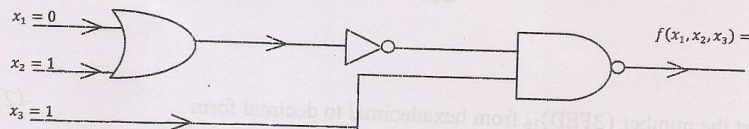
Q.4

a. Let $L_1 = \{x, xy, xy^2, xy^3\}$ and $L_2 = \{x, xyx\}$, then find L_1^2 , L_1L_2 and L_2^2 (3)

b. Prove that $(a + b) \cdot (\bar{b} + c) + b \cdot (\bar{a} + \bar{c}) = a \cdot \bar{b} + a \cdot c + b$ by using the properties of Boolean algebra (3)

c. Find the output for a given input from the following circuit

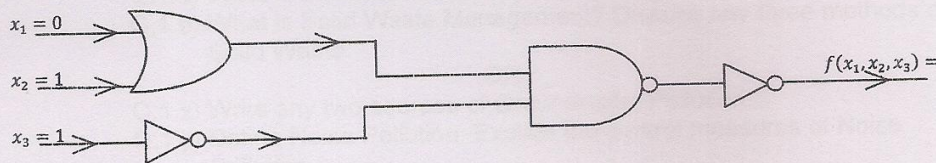
Inputs $x_1 = 0, x_2 = 1$ and $x_3 = 1$ (4)



OR

Q .IV

- A. Find the language $L(G)$ generated by grammar with $v = \{S, A, B\}$, $\sigma = \{a, b\}$ and production $P = \{S \rightarrow aB, B \rightarrow b, B \rightarrow bA, A \rightarrow aB\}$ (3)
- B. Prove that $x \cdot x = x$ and $x + x = x$ by using the properties of Boolean algebra (3)
- C. Find the output for a given input from the following circuit (4)
- Inputs $x_1 = 0, x_2 = 1$ and $x_3 = 1$



Q .5

- a. In how many different ways can a six letter word be formed using the letters N, U, M, B, E, R (1)
- b. Find r if ${}^8P_r : {}^6P_r = 56 : 3$ (3)
- c. Define the following terms with examples (3)
- i. Null set
 - ii. Singleton set
 - iii. Cartesian product of two non-empty sets
- d. Find $f \circ g$ and $g \circ f$ for the following functions (3)
- $f(x) = \frac{4x-1}{4x+1}$ and $g(x) = x - 2$

OR

Q .V

- A. If $A = \{a, b, c, d\}$ and $B = \{b, d, e\}$ find (3)
- i. $(A \cap B) \times (A - B)$
 - ii. $A \times (A - B)$
 - iii. $(A - B) \cup (B - A)$
- B. Verify whether the function $f : \mathbf{R} \longrightarrow \mathbf{R}$ defined by $f(x) = 2x - 3$ is a bijection. Then find its inverse function (3)
- C. A committee of 10 is formed from 5 women and 7 men. In how many ways can this be done if the committee must contain atleast 3 women? (4)

XXXXXXXXXXall the bestXXXXXXXXXX