Duration: 2 hours   Marks: 50

Instructions:
1. All questions are compulsory. However internal choice has been provided for Q.2 - Q.5
2. Figures to right indicate full marks.
3. Use of non-programmable calculators are allowed.

Q 1) Answer the following.   **(10×1=10)**
   (a) If $X = \{1,2,3,4\}$ and $A = \{2,3,4\}$ then $A^C =$ _____.
   (b) If A= {a} and B={1,3} then A×B=_____.
   (c) L^+ is given by the formula _____.
   (d) $P_3^5 =$ _____.
   (e) If $f(x) = 2x$ and $g(x) = logx$, then $(f \circ g)(x) =$ _____.
   (f) Define string over an alphabet σ.
   (g) Base for hexadecimal system is _____.
   (h) $(72)_{10}$ is equivalent to _____ in binary form.
   (i) The logic symbol for AND gate is _____.
   (j) Idempotent law on a Boolean algebra is given by _____.

Q 2) Answer the following.
   (a) Convert $(3FED)_{16}$ to its decimal equivalent.   **(3)**
   (b) Prove that relation R on the set of integers $\mathbb{Z}$ defined as   **(5)**
       $R = \{(x, y)|x - y \text{ is divisible by } 3, \ x, y \in \mathbb{Z}.\}$
   (c) If $f(x) = \frac{2x+1}{2x-1}$ then find $f(3x)$.   **(2)**

                                         OR

   (d) Convert $(643.25)_{10}$ to binary fractions.   **(3)**
   (e) Let $A = \{1,2,3\}$ and a relation on A be   **(5)**
       R={(1,1), (1,2), (2,1), (2,2), (2,3), (3,3)}. Prove that the relation R is reflexive, not symmetric and not transitive.
   (f) If $f(x) = 3x - 1$ & $g(x) = x^2 + 1$ find $f(g(x))$.   **(2)**

Q 3) Answer the following.
   (a) Prove that $(p \to q) \equiv [(\sim q) \to (\sim p)]$.   **(4)**
   (b) Find the coefficient of x^2y^3 in the expansion of $(2x + y)^5$.   **(3)**

                                                                                          P.T.O.

(c) Prove by mathematical induction that (3)
$$1 + 3 + 6 + \cdots + \frac{n(n+1)}{2} = \frac{n(n+1)(n+2)}{6}$$

OR

(d) Find the truth values for $(p \vee q) \leftrightarrow [q \vee (r \rightarrow p)]$. (4)

(e) Find the 5th term in the expansion of $(2x^2 + \frac{3}{2x})^7$. (3)

(f) Prove by mathematical induction that
$$1^2 + 2^2 + 3^2 + \cdots + n^2 = \frac{n(n+1)(2n+1)}{6}$$ (3)

Q 4) Answer the following.

(a) A committee of 10 is to be formed from 5 women and 7 men. In how many ways this can be done if committee contains

     (i)   3 women

     (ii)   At most 2 men (5)

(b) Define finite state machine and write down its 3 major features. (5)

OR

(c) A club has 5 girls and 7 boys. If 4 persons out of these to be selected, find out the total number of choices if

     (i)   2 boys and 2 girls are to be selected

     (ii)   At most 1 girl is to be selected (5)

(d) Find the language L(G) generated by the grammar with $V = \{S, A\}$ & $\sigma = \{x, y, z\}$ and production $P = \{S \rightarrow xSy, xS \rightarrow Ay, Axy \rightarrow z\}$ (5)

Q 5) Answer the following.

(a) If $A = \{a, b, c, d\}$ & $B = \{b, d, e\}$ find

     (i) $(A \cap B) \times (A - B)$

     (ii) $A \times (A - B)$

     (iii) $(A - B) \cup (B - A)$ (5)

(b) Draw the symbol for XOR gate and write its truth table. (5)

OR

(c) If $X = \{A, B, C, D, E, G, H, I, L, M, N, O, R, S, T, Y, Z\}, B = \{T, H, E, O, R, Y\}$ & $C = \{T, H, E, O, R, M, S\}$ then verify that

     (i) $B - C = (B^C \cup C)^C$

     (ii)   Find $A \cap B$ (5)

(d) Write the truth table for NAND gate and draw its symbol. (5)

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