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B.C.A (SEMESTER-I) EXAMINATION, OCTOBER 2015
BCA 104 BASIC MATHEMATICS

Duration : 2 hours Marks: 50

Q.1 Fill in the blanks: (10x1 = 10)

a) $(\log_b a)(\log_a b) =$ ………… where, $a > 1, b > 1$

b) If $5^a = 625$ then, $a =$ …………

c) Area of a circle of radius $'r\ cm'$ is given by ……….. cm^2

d) If a, b, c are in arithmetic progression, then b = …………..

e) Let $z = 3 + 4i$, then $\bar{z} =$ ………………

f) If $f(x) = x^3$, then $f(\log x) =$ …………

g) If 4:7:: x :35, then $x =$ ……….,

h) The factors of $x^2 + 3x + 2$ are ……….. and …………..

i) The greatest common divisor (g.c.d) of 37 and 249 is………

j) If $\log_2 128 = x$, then $x =$ ………

Q.2

A. Prove that the vectors $\vec{a} = \hat{\imath} + 2\hat{\jmath} + \hat{k}$ and $\vec{b} = \hat{\imath} + \hat{\jmath} + 3\hat{k}$ are perpendicular to each other. (2)

B. Find the length of the canvas 2 metres in width required to make a conical tent 8 meters in diameter & 5.6 metres in slant height. (3)

C. Without actual expansion as far as possible prove the following (5)

$$\begin{vmatrix} 1 & x & x^2 \\ 1 & y & y^2 \\ 1 & z & z^2 \end{vmatrix} = (x-y)(y-z)(z-x)$$

OR

Q.II

a. Find the area of the parallelogram whose adjacent sides are given by vectors $\vec{a} = \hat{i} - 2\hat{j} + 3\hat{k}$ and $\vec{b} = 3\hat{i} - 2\hat{j} + \hat{k}$ (2)

b. The side of a square field is 89 metres. By how much square metre does its area fall short of hectare?
(Given: A hectare = 10000 mt^2) (3)

c. Solve the following system of equations by using matrix method (5)

$5x + 3y + z = 16$

$2x + y + 3z = 19$

$x + 2y + 4z = 25$

Q.3

A. Find $\vec{a} \times \vec{b}$ if $\vec{a} = 2\hat{i} + 3\hat{j} + 6\hat{k}$ and $\vec{b} = 3\hat{i} - 6\hat{j} + 2$ (2)

B. Find the sum of all the numbers between 100 and 400 which are exactly divisible by 3. (3)

C. Evaluate the following limit

$$\lim_{x \to 2} \left[\frac{1}{x^2 + x - 6} + \frac{1}{x^2 - 9x + 14} \right]$$ (5)

OR

Q.III

a. Find a unit vector perpendicular to both the vectors $\vec{a} = 4\hat{i} - \hat{j} + 3\hat{k}$ and $\vec{b} = -2\hat{i} + \hat{j} - 2\hat{k}$ (2)

b. If a, b, c are in A.P, prove that

$3a^2 - 4b^2 + c^2 = 2a(a - c)$ (3)

c. Discuss the continuity of the following function at the point indicated

$$f(x) = \begin{cases} \sin x & , 0 \le x \le \pi/4 \\ \tan x & , \pi/4 < x \le \pi/2 \\ \cos x & , \pi/2 < x \le 3\pi/4 \end{cases}$$

At $x = \pi/4$ and $x = \pi/2$ (5)

Q.4

A. Using trigonometry, prove the identity (3)

$$\sin 2\theta = \frac{2\tan\theta}{1+\tan^2\theta}$$

B. Use De Moivre's theorem to prove the following (3)

$\sin 2\theta = 2\sin\theta\cos\theta$

C. Find m and n if $(m, n+1)$ divides segment AB externally in the ratio 2:1 where A = $(-3,1)$ and B=$(-6,7)$ (4)

OR

Q.IV

a. Using trigonometry, prove the following identity (3)

$\cos 2\theta = 1 - 2\sin^2\theta$

b. Use De Moivre's theorem to prove the following (3)

$\cos 3\theta = 4\cos^3\theta - 3\cos\theta$

c. Find the equation of the line through the point of intersection of

x + 2y - 4 =0, x – 3y + 1 =0 and also through the mid-point of the segment joining (2,5) and (4,3) (4)

Q.5

A. If $f(x) = a\sin(\log x)$, prove that

$x^2 f''(x) + xf'(x) + f(x) = 0$ (5)

B. Evaluate $\int_0^{\log 3} \frac{e^x}{1+e^x}dx$ OR

Q.V

a. Show that f(x)= $x^2 - 9x^2 + 30x + 5$ has neither maxima nor Minima. (3)

b. Differentiate y = $(x^2 - 3x + 5)^{10}$ with respect to x (2)

c. Prove that the area bounded by the curve $y = x^2 - 3x$ and the line $y = 2x$ is $\frac{125}{6}$ square units (5)

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